Reply to "Comment on 'Existence and design of trans-vacuum-speed metamaterials' "

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It is clarified further mathematically that the two-time derivative Lorentz metamaterial (2TDLM) model, which was used to achieve the trans-vacuum-speed effects in [Phys. Rev. E **68**, 026612 (2003)], has a delayed causal response in agreement with the analytical and numerical results given there. The key point that the analytical (explicit low-pass filtered), and the numerical finite difference time domain and circuit simulator (implicit low-pass filtered) results for information propagation in a nonperfect 2TDLM medium still exhibit the trans-vacuum-speed (TVS) effect, despite the (physically required) imposed absence of the 2TDLM's unusual high-frequency behavior, is also discussed in more detail.

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I would like to thank the author for all of his comments (preceding paper [1]), but I do not believe that they disprove the analytical or numerical results for the perfect or the nonperfect trans-vacuum-speed (TVS) metamaterial media given in either [2] or [3].

Having agreed with the fact that the two-time derivative Lorentz metamaterial (2TDLM) model is causal and satisfies the extended Kramer-Kröning relations derived in [2], the author's arguments contradict the general analysis given, for instance, in [4]. In particular, since from [2] we have $\varepsilon(\omega) = \varepsilon_0[1+\chi(\omega)]$ and $\varepsilon_\infty = \varepsilon_0[1+\chi(\infty)]$, Jackson's expressions can be readily rewritten as

$$D(t) = \varepsilon_\infty E(t) + \varepsilon_0 \int_0^t d\tau G_\infty(t-\tau) E(\tau), \tag{1}$$

$$\frac{\varepsilon(\omega) - \varepsilon_\infty}{\varepsilon_0} = \chi(\omega) - \chi(\infty) = \int_0^\infty G_\infty(\tau) e^{-j\omega\tau} d\tau, \tag{2}$$

$$G_\infty(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\chi(\omega) - \chi(\infty)] e^{+j\omega\tau} d\omega. \tag{3}$$

With (1)–(3) the correct extension of the last equation on page 309 of [3] to the present discussion is

$$\lim_{\omega\to\infty} [\chi(\omega) - \chi(\infty)] = -j\frac{G_\infty(0+)}{\omega} - \frac{G'_\infty(0+)}{\omega^2} + \cdots, \tag{4}$$

and not (5) of the Comment. For the 2TDLM medium one finds $\chi(\infty) = \chi_\gamma$; we then have for large frequencies

$$\lim_{\omega\to\infty} [\chi(\omega) - \chi(\infty)] = \lim_{\omega\to\infty}\left(\frac{\omega_p^2\chi_\alpha + j\omega\omega_p\chi_\beta - \omega^2\chi_\gamma}{-\omega^2 + j\Gamma\omega + \omega_0^2} - \chi_\gamma\right) \approx -j\frac{(\omega_p\chi_\beta - \Gamma\chi_\gamma)}{\omega} - \frac{(\omega_p^2\chi_\alpha - \omega_0^2\chi_\gamma)}{\omega^2}. \tag{5}$$

As shown by (16) of [2] for large frequencies, the medium is passive if the term $G_\infty(0+) = \omega_p\chi_\beta - \Gamma\chi_\gamma > 0$ and, hence, the sign of the leading term in (5) is in agreement with Jackson's arguments for passivity. Moreover, even though this leading term is nonzero, the medium response is not instantaneous. This type of high-frequency response has been addressed by many authors, for instance in [4–6], in regards to basic lossy dispersive media. Vekstein describes the process of handling this behavior in a convenient manner: the permittivity expression is "regularized" to deal with the conductive loss term. The final mathematical results, i.e., the Kramers-Krönig relation, the displacement relation (1) (Eq. (4) in [2]), and the Green's function for such a medium, are all self-consistent and show that no instantaneous response will occur. As discussed in [2], the requirement that $\chi_\gamma > -1$ ensures causality and no instantaneous response (i.e., the response at the observation point would be instantaneous iff $\chi_\gamma \equiv -1$). By combining the conduction type current term, i.e., the leading term in (5), together correctly with the displacement current component, the proper delayed, causal time response is obtained. This behavior also holds true for the TDLM medium discussed in [7], i.e., the 2TDLM medium with $\chi_\gamma = 0$. With $\chi_\gamma = 0$ the leading term in (4) is still nonzero. The Green's function for the TDLM medium was obtained in [7]; and it was shown that a causal, properly delayed response occurs. The lossy dispersive Debye model exhibits a similar behavior. It was shown by Roberts and Petropoulos in [8] that the Green's function for a Debye medium is causal and produces a retarded response.

Finally, Roberts has proven with full mathematical generality two issues in [9] that have a definitive impact on this discussion. In particular, removing the spatial dependence for the nondispersive parts of the permittivity and conductivity in his model medium, the resulting medium encompasses the 2TDLM model including the high-frequency response ε_∞ and the "loss" component $G(0^+)$. Using the equivalent of (5), he proved conclusively that the response of this medium is causal independent of the value of ε_∞. Moreover, he showed that the response is retarded, i.e., it is not instantaneous. It thus agrees with the Kramers-Krönig conclusions in [2] and the resulting delayed responses demonstrated in [2,3].

Consequently, in contrast to the Comment's suggestions, the delayed responses for the 2TDLM medium discussed in [2,3] appear to agree with several recent and past studies of causality and delayed responses in general lossy, dispersive media.

The final discussion of the comment concerning the physical (not mathematical) issue of the local versus global validity of the metamaterial models and the practicality of realizing the infinite frequency behavior in any realistic situation actually reinforces similar comments made in [2,3]. In particular, this fact stimulated the nonperfect TVS medium investigations in both [2,3]. The Comment's author may have missed the numerical subtlety that the finite difference time domain (FDTD) grid itself acts implicitly as a low-pass filter. Thus, despite the unusual infinitely high-frequency behavior of the material models considered in [3], the numerical results do not include it. This was made explicit in [2] by including the low-pass filter to generate the nonperfect TVS medium results. Even if there were some type of dynamic pulse reshaping occurring near the front (infinite frequency) portion of the propagating pulse in the global model, it is not observed in the nonperfect case because these components are completely filtered out of the information signal. In both [2,3] the TVS behavior was maintained even when the 2TDLM response was localized to lower frequencies, i.e., the TVS behavior is maintained even in the nonperfect 2TDLM medium. This observation is further reinforced by the frequency dependent and independent lumped element transmission line results shown in [2]. The TVS transmission line produced the TVS effect as long as the unit-cell sizes are much smaller than the characteristic lengths of the information signal. Because the highest frequencies correspond to yet even smaller distances, they would not be captured by a fixed-size unit cell; i.e., the unit cell would act as a low-pass filter like the FDTD mesh and, hence, these highest frequency components would be lost from the signal or turned into noise. Nonetheless, because the design of the information pulses in [2,3] causes the amount of signal energy carried by these high-frequency components to be negligible in comparison to that carried by the main portion of the information signal at lower frequencies, the loss of the high-frequency components has little effect on the overall propagation characteristics of that information signal.

Thus, as demonstrated in [2,3], the TVS effect remains even when, as pointed out in the Comment, the high-frequency components are not accessible for physical reasons other than those associated with causality or time retardation.

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